MULTI-CARRIER MICROWAVE BREAKDOWN IN AIR-FILLED COMPONENTS

Ulf Jordan[1], Torbjörn Olsson[2], Vladimir E. Semenov[3], Dan Anderson[1], and Mietek Lisak[1]

[1]Department of Electromagnetics, Chalmers University of Technology,
SE-412 96 Göteborg, Sweden
[2]Allgon Systems AB, Antennvägen 6, SE-187 80 Täby, Sweden
[3]Institute of Applied Physics, Russian Academy of Sciences, 46 Ulyanov Street, Nizhny Novgorod,
603 600 Russia

***Abstract* – An investigation is made of the threshold for microwave breakdown in mobile telephone communication systems based on multi-carrier operation where interference between the carriers may cause occasional high power peaks in the transmission system. Thresholds are established for the worst case scenario of coherent and co-phased carriers as well as for the more realistic case of modulated carriers when the signal is considered as a stochastic signal.**

I. INTRODUCTION

The transfer of information in many modern communication systems is often accomplished by using multi-carrier operation, especially to extend systems with more information channels. The information is kept separated by using carrier waves of different frequencies cf wavelength and frequency multiplexing in optical fibres and in mobile telephone communication systems.

Multi-carrier operation in mobile telephone communication systems is often based on frequency equidistant carriers. For coherent and co-phased carrier waves, the signals will, with a repetition frequency determined by the channel separation, add constructively to give a resulting amplitude which is much larger than the amplitude of a single carrier. Obviously this increases the liability for microwave breakdown and it becomes necessary to assess the importance of this phenomenon for the breakdown strength of the system. Important factors in this context are expected to be e. g. (i) the amplitude and (ii) the time duration of the high power peak, (iii) the repetition frequency of the peaks, and (iv) possible intentional or unintentional de-phasing or de-correlation between different carrier waves. A detailed investigation of these effects will be given in the following study.

Breakdown conditions for the worst case scenario of a number of coherent co-phased carriers is derived using a modeling of the signal as a sequence of periodically appearing rectangular pulses. In this model, the pulse repetition frequency, the pulse length and the peak amplitude are determined by the amplitude of the individual carriers together with the number of carriers and the frequency separation between the different channels. In order to analyze the complicated signal resulting from many carriers all transmitting information, the signal is then considered as a stochastic process and the stochastic properties of breakdown in this situation are also analyzed. The result is then obtained in terms of the probability of breakdown.

Obviously a conservative approach to avoid breakdown in multi-carrier systems is to design the system so that the high peak power replaces the single carrier power level in the breakdown calculations. However, this approach tends to be too conservative since it neglects the finite time duration of the high power peak, which for large values of the number of carriers, i. e. short high power peaks, can be expected to lead to less stringent conditions on the breakdown thresholds. An analysis of the effects of finite pulse length, τ_p , and pulse repetition frequency $f_{\mathrm{PRF}} = 1/T$ on the breakdown condition is given in section III after a short review of relevant breakdown results has been given in section II.

II. MICROWAVE BREAKDOWN IN AIR: BASICS

We begin this section by rapidly summarizing relevant results for microwave breakdown in air at atmospheric pressures where the dominant loss mechanism for the free electrons is attachment on oxygen atoms and molecules. Under these conditions the breakdown phenomenon can be analyzed using the following simple local equation for the electron density:

$$\frac{\partial n}{\partial t} = \nu_i(t)n - \nu_a(t)n + J \tag{1}$$

where ν_i is the frequency of electron impact ionization, ν_a is the frequency of electron attachment, and J is a constant source of free electrons due to background radiation. Both ν_i and ν_a depend on microwave power, P, in a way which can be approximated as

$$\nu_i - \nu_a \approx \begin{cases} \nu_h \left[\left(\frac{P}{P_b^{CW}} \right)^{\beta} - 1 \right] & \text{when } P \approx P_b^{CW} \\ -\nu_c & \text{when } P \ll P_b^{CW} \end{cases} \quad (2)$$

In (2), $\beta \approx 2.67$, P_b^{CW} is the microwave breakdown threshold for constant amplitude CW operation, which depends on the details of the microwave system used, the characteristic frequency ν_h is $\nu_h \approx 6 \cdot 10^4 p$ (where ν_h is in s^{-1} and the pressure, p, is in Torr), and finally $\nu_c \approx 100 p^2$ is the frequency of three-body attachment of cold electrons (where again ν_c is in s^{-1} and p is in Torr), cf [1], [2].

A. CW Breakdown

For constant microwave power, (1) has a stable steady state solution $n_s = J/(\nu_a - \nu_i)$ if $P < P_b^{CW}$ whereas n increases exponentially $n \propto \exp(\gamma t)$ with constant growth rate $\gamma = \nu_i - \nu_a$ if $P > P_b^{CW}$. In the latter case, the electron density will reach high values and breakdown occurs. In principle, high electron densities can be attained even if P is less than but sufficiently close to P_b^{CW}. However the source J is usually so weak that from a practical point of view, the breakdown threshold for microwave power in the CW regime can be taken as $P = P_b^{CW}$ corresponding to $\nu_i - \nu_a = 0$.

B. Breakdown in the regime of periodic pulse repetition

When ν_i and ν_a are periodic functions of time, the character of the solution given by (1) depends on the sign of the average value of the difference $\nu_i - \nu_a$ denoted $\langle \nu_i - \nu_a \rangle_T$. Similarly to the case in the CW regime, the solution increases without bounds when $\langle \nu_i - \nu_a \rangle_T > 0$ while there is a stable finite periodic solution if $\langle \nu_i - \nu_a \rangle_T < 0$. However in contrast to the CW regime, the peaks of the electron density can become very high in the periodic regime even if $\langle \nu_i \rangle_T$ is well below $\langle \nu_a \rangle_T$. Specifically, using the simplest rectangular approximation for the time variation of the microwave pulse power we obtain $\nu_i - \nu_a = \gamma =$ const. within the temporal interval τ_p, and $\nu_i - \nu_a = -\nu_c$ during the remaining part $\tau_c = T - \tau_p$ of the period. One can then easily find the maximum value of the electron density variation as follows:

$$n_{\max} = n_0 e^{\gamma \tau_p} \frac{1 - e^{-\nu_c \tau_c} + (1 - e^{\gamma \tau_p}) \nu_c / \gamma}{1 - e^{(\gamma \tau_p - \nu_c \tau_c)}} \quad (3)$$

where $n_0 = J/\nu_c$ is a small background electron density in the air. The above expression for maximum electron density, $n_{\max}$, makes it possible to determine the threshold value of γ (i. e. the threshold power of the microwave pulse) by comparing $n_{\max}$ with the critical electron density n_c at which the breakdown plasma has significant effects (in the form of reflection and/or absorption) on the microwave propagation. A simple estimate of n_c is that the corresponding plasma frequency equals the microwave frequency. It should be noted that previously discussed breakdown criteria for CW, single-, and multi-pulse cases (e. g. [1]-[3]) are contained in (3). Equations (2) and (3) also define a relationship between pulsed and CW breakdown thresholds, which may be important in validation experiments using CW and pulsed operation respectively. In [4], a CW experiment at reduced pressure is described. However, usually one needs, for thermal reasons, to switch to pulsed operation at normal pressures.

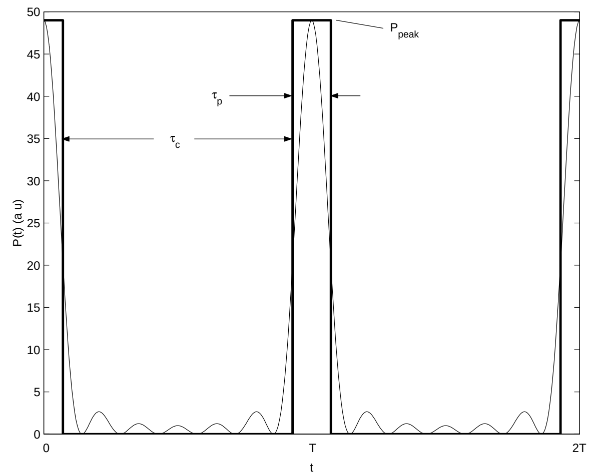

Fig. 1: Example of rectangular pulse approximation of the power of the co-phased multi-carrier signal.

III. Deterministic Breakdown Threshold

From the point of view of breakdown, the worst case multi-carrier scenario arises for co-phased carriers. Consider $2N+1$ carriers of equal power, P_c, and with a frequency separation Δf. In this case peak power, P_{peak}, time duration of pulse peak, τ_p, and time between pulses, τ_c, are given by

$$\begin{aligned} P_{\text{peak}} &= (2N+1)^2 P_c \\ \tau_p &\approx \frac{1}{(2N+1)\Delta f} \\ \tau_c &\approx \frac{1}{\Delta f} \frac{2N}{2N+1} \end{aligned} \quad (4)$$

When $N \gg 1$, the multi-carrier signal can be modeled as a train of pulses, each of power P_{peak} and duration τ_p and separated by the time τ_c, cf Fig. 1.

Within such a model the breakdown threshold can be estimated from (2) and (3) using the condition

$$\ln \frac{n_{\max}}{n_0} = \ln \frac{n_c}{n_0} \approx 20 \quad (5)$$

as the breakdown criterion, cf [1], [2]. This gives an implicit equation for P_c/P_b^{CW} or, with $\langle P\rangle = (2N+1)P_c$, for $\langle P\rangle/P_b^{CW}$ which is readily solved numerically, yielding the deterministic breakdown level. If the individual carriers are modulated with a peak-to-average-power factor PF, the highest obtainable peak power is PF larger than in the umodulated case. For a conservative estimate of the breakdown threshold the permissible average power of the multi-carrier signal must consequently be lowered by the factor PF.

We emphasize that the present analysis of the electron density evolution is based on the assumption that in the discharge region there is always some finite number of free electrons available to start the breakdown avalanche. However, in situations involving strongly inhomogeneous electric fields, the region where the electron avalanche is initiated can become very small. In this case, the active region may have a stochastically varying number of initial electrons including zero during some temporal intervals. In such cases, the stochastic waiting time for the first electron will become important.

IV. Breakdown Threshold: Stochastic Signal

We will now generalize the previous analysis to account for a more realistic signal variation. Since the actual signal formats and codings in realistic communication systems can be very complicated, a good first order approach would be to consider the signal as stochastic.

We model the amplitude of the multi-carrier signal as a stationary Gaussian process with zero expectation value and known auto-correlation function. This leads to the power of the signal being described by

$$\begin{aligned} \langle P(t)\rangle &= \langle P\rangle \\ \langle P(t+\tau)P(t)\rangle &= \langle P\rangle^2 + \langle P\rangle^2 R_p(\tau) \end{aligned} \tag{6}$$

where $\langle P\rangle$ is the average power of the signal and $R_p(\tau)$ is the power auto-correlation function. The probability distribution over power becomes

$$dP_P = \frac{1}{\langle P\rangle}\exp\left(-\frac{P}{\langle P\rangle}\right)dP \tag{7}$$

and the power correlation time $\tau_p \approx 1/\Delta F$ is estimated similarly to (4) from the total bandwidth, ΔF, of the multi-carrier signal. The breakdown condition for single pulse breakdown is taken as 20 e-foldings, cf [1], [2]:

$$(\nu_i(P_b) - \nu_a(P_b))\tau_p \approx \ln\frac{n_c}{n_0} \approx 20 \tag{8}$$

which gives the single pulse breakdown threshold, P_b. The probability for a pulse to cause breakdown is then

$$P\left(P(t) > P_b\right) = \int_{P_b}^{\infty} dP_P = \exp\left(-\frac{P_b}{\langle P\rangle}\right) \tag{9}$$

Considering breakdown due to single high power peaks (provided that $\langle P\rangle \ll P_b^{CW}$), the fraction of time with high plasma density can be estimated as, assuming there are free electrons available to start the avalanche from and that the breakdown level for subsequent events is not changed,

$$R \approx \frac{t_b}{\Delta t_b} \tag{10}$$

Here t_b is the expectation value of the time duration of high plasma density calculated over the distribution of high power peaks ($P > P_b$) according to the probability density distribution (7) assuming that the electron density saturates at the critical density, n_c, if reached before τ_p, and Δt_b is the expectation value of the time between the high power peaks, estimated to be

$$\Delta t_b \approx \frac{\tau_p}{P\left(P(t) > P_b\right)} = \tau_p \exp\left(\frac{P_b}{\langle P\rangle}\right) \tag{11}$$

We obtain the following approximate expression for the fraction R:

$$\begin{aligned} R \;\approx\; & \frac{1}{\nu_c\tau_p}\exp(-\frac{P_b}{\langle P\rangle}) \times \\ & \{1 + \nu_c\tau_p \cdot \min[1, \frac{1}{20} + \beta\frac{\langle P\rangle}{P_b}(1 + \frac{\nu_h\tau_p}{20})]\} \end{aligned} \tag{12}$$

V. Application: The Derating Function

When designing a multi-carrier communication system, it is important to analyze in which way the system is limited by different constraints when varying system parameters, in particular the number of carriers. We will here consider two constraints; the total average power of the multi-carrier signal (a limitation set e. g. by thermal breakdown considerations) and the microwave dielectric breakdown threshold. For the microwave breakdown threshold, the deterministic implicit conditions ((2), (3), and (5)) as well as the stochastic constraint (12) are taken into account, the latter by keeping R at a constant specified level during variation of the number of carriers.

In Fig. 2 we show the "derating function" i. e. the total average power $\langle P\rangle$ of the multi-carrier signal (compared to the CW breakdown threshold, P_b^{CW}) as a function of the number of carriers for typical D-Amps parameters ($\Delta f = 630$ kHz and $PF = 2$). The constraint of constant average power ("the thermal constraint") would in such a graph appear as a horizontal line. If we assume this level to be larger than the CW breakdown level, the thermal constraint will not be important for the system design. The constraint corresponding to the "worst case" deterministic threshold for co-phased carriers is given by the solid line in Fig. 2 for high number of carriers. It represents a quite severe limit on the permissible power. However, on the other hand it guarantees that no

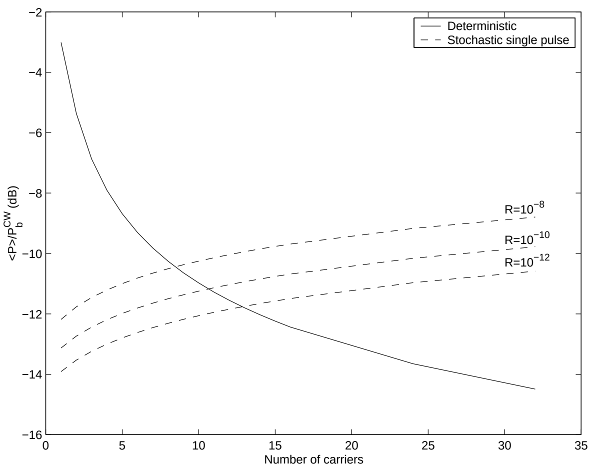

Fig. 2: The derating function $\langle P\rangle/P_b^{CW}$ as a function of number of carriers. The solid line gives the determinsistic limit, and the dashed lines are for constant fractional breakdown times $R=10^{-8}$, $R=10^{-10}$, and $R=10^{-12}$.

signal with a power below this limit will ever cause breakdown. In a real design it will of course be necessary to add a safety factor to the deterministic limit, since it represents a limiting value.

If we turn to the constraint determined by the figure of merit from the stochastic treatment, a completely different picture emerges. In this example, the constraining constant values are $R=10^{-8}$, $R=10^{-10}$, and $R=10^{-12}$. One interesting feature is that there is a slight increase in the permissible average power for increasing number of carriers. The physical origin of this property is the decrease in the correlation time, τ_p, with increasing number of carriers.

Note that there is a region in de-rating space which is bounded from above by the constant R curves and from below by the deterministic breakdown threshold condition. In this region, the possibility of breakdown can not be excluded from a deterministic point of view. The extension of this region depends on the chosen values of R and the region can be made very narrow by choosing R small. On the other hand, from a practical point of view, this “shadow” zone of operation might still be usable, provided occasional breakdown can be accepted. Since R is the fractional time with high plasma density it can be used in an attempt to precisely quantify the consciously compromised level of security against microwave breakdown in the design of a component used in a communication system. However, a more detailed investigation would be needed in order to assess an acceptable level of breakdown probability and in particular the consequences of a breakdown event. As an example, it is well known that the microwave power needed to sustain a breakdown plasma is smaller than that needed to initiate breakdown. Consequently, does a breakdown induced plasma vanish, once it has been initiated, or will it be sustained by the microwave signal for long periods of time, thus significantly degrading normal operation? At what fraction of time, R, is the impact on the gas sufficiently small, as not to reduce the threshold for subsequent high power peaks? It is known that, for instance a changed gas composition reducing the threshold level, may result from a plasma discharge.

Summary

A detailed analysis has been made of the critical conditions for breakdown in multi-carrier systems where interference between the carriers may cause occasional high power peaks to occur in the transmission system. An analysis is made of the worst case scenario of coherent and co-phased multi-carrier signals. The result provides limits beyond which breakdown should not occur and can be used as a conservative design rule, although the conditions tend to be too restrictive. In order to make a better model of a realistic information-carrying signal we then consider the multi-carrier signal as a stochastic process where signal statistics can be used to predict the breakdown properties of a component. In this work the multi-carrier signal is modeled as a Gaussian process. The stochastic approach gives results in terms of the average time that a component is affected by breakdown, if a background of free electrons is available as assumed.

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